Bondal-Orlov's reconstruction theorem in noncommutative projective geometry

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Research Interests:

- Noncommutative Algebraic Geometry (NC AG) /Generalized complex geometry (GCG)
- Moduli spaces of sheaves, Derived categories
- Calabi-Yau manifolds, Mirror symmetry

Hello!
こんにちは！
("kon-ni-chi-wa")

A Quick Introduction to NC Proj Geometry

- $k = \overline{k}$.
- $R = \bigoplus_{i \in \mathbb{N}} R_i$: commutative $\mathbb{N}$-gr k-alg with $R_0 = k$
- $\mathrm{qgr}(R) = \mathrm{gr}(R)/\mathrm{tor}(R)$: quotient category.

Theorem (Gabriel '62)

X, Y : noeth schs.
Then,

$$\mathrm{Coh}(X) \simeq \mathrm{Coh}(Y) \Rightarrow X \simeq Y.$$

Theorem (Serre '55)

Assume that R is generated by R_1 as a k-alg.
Then,

$$\mathrm{Coh}(\mathrm{Proj}(R)) \simeq \mathrm{qgr}(R).$$

Important

- **qgr(R) characterizes Proj(R)!**
- **qgr can also be defined for noncommutative graded rings!**

A Recent Result

Theorem (Bondal-Orlov '01)

X, Y : smooth proj vars over k.
Assume that the canonical bundles of X, Y are (anti-)ample.
Then,

$$D^b(\mathrm{Coh}(X)) \simeq D^b(\mathrm{Coh}(Y)) \Rightarrow X \simeq Y.$$

- $A = \bigoplus_{i \in \mathbb{N}} A_i$: noetherian $\mathbb{N}$-gr k-alg with $A_0 = k$.
- $\mathrm{qgr}(A) = \mathrm{gr}(A)/\mathrm{tor}(A)$: the NC proj scheme associated with A.

Theorem (M '24)

Assume a condition ($^{\exists}$balanced dualizing complexes of A, B).
If "the canonical bundles" of $\mathrm{qgr}(A), \mathrm{qgr}(B)$ are "(anti-)ample", then
Bondal-Orlov reconstruction holds for NC proj schs:

$$D^b(\mathrm{qgr}(A)) \simeq D^b(\mathrm{qgr}(B)) \Rightarrow \mathrm{qgr}(A) \simeq \mathrm{qgr}(B).$$

Question

What about the case of NC proj Calabi-Yau mfds?

Rmk

In previous works, Kanazawa and I provided examples of NC CY mfds as NC analogues of hypersurfaces of weighted proj sps.

Thank you for your attention!